

the breaking-weight upon bridges of this description, as, according to the last experiment, the beam broke with 313,000 changes; or a period of eight years, at the same rate as before, would be sufficient to break it. It is more than probable that the beam had been injured by the previous 3,000,000 changes to which it had been subjected; and assuming this to be true, it would follow that the beam was undergoing a gradual deterioration which must some time, however remote, have terminated in fracture.

February 11, 1864.

Major-General SABINE, President, in the Chair.

The following communications were read:—

- I. "On the Calculus of Symbols.—Fourth Memoir. With Applications to the Theory of Non-linear Differential Equations." By W. H. L. RUSSELL, A.B. Communicated by Professor CAYLEY. Received July 31, 1863.

(Abstract.)

In the preceding memoirs on the Calculus of Symbols, systems have been constructed for the multiplication and division of non-commutative symbols subject to certain laws of combination; and these systems suffice for linear differential equations. But when we enter upon the consideration of non-linear equations, we see at once that these methods do not apply. It becomes necessary to invent some fresh mode of calculation, and a new notation, in order to bring non-linear functions into a condition which admits of treatment by symbolical algebra. This is the object of the following memoir. Professor Boole has given, in his 'Treatise on Differential Equations,' a method due to M. Sarrus, by which we ascertain whether a given non-linear function is a complete differential. This method, as will be seen by anyone who will refer to Professor Boole's treatise, is equivalent to finding the conditions that a non-linear function may be externally divisible by the symbol of differentiation. In the following paper I have given a notation by which I obtain the actual expressions for those conditions, and for the symbolical remainders arising in the course of the division, and have extended my investigations to ascertaining the results of the symbolical division of non-linear functions by linear functions of the symbol of differentiation.

- II. "On Molecular Mechanics." By the Rev. JOSEPH BAYMA, of Stonyhurst College, Lancashire. Communicated by Dr. SHARPEY, Sec. R.S. Received January 5, 1864.

The following pages contain a short account of some speculations on molecular mechanics. They will show how far my plan of molecular

mechanics has been as yet developed, and how much more is to be done before it reaches its proper perfection. Of course I can do no more than point out the principles on which, according to my views, this new science ought to be grounded. The *proofs* would require a volume,—and the more so, as existing wide-spread philosophical prejudices will make it my duty to join together both demonstration and refutation. But there will be time hereafter, if necessary, for a complete exposition and vindication of the principles on which I rely; at present it will be enough for me to state them.

The aim of “molecular mechanics” is the solution of a problem which includes all branches of physics, and which may be enunciated, in general terms, as follows :—

“From the knowledge we gain of certain properties of natural substances by observation and experiment, to determine the intrinsic constitution of these substances, and the laws according to which they ought to act and be acted upon in any hypothesis whatever.”

In order to clear the way for the solution of this problem, three things are to be done.

First. From the known properties of bodies must be deduced the essential principles and intrinsic constitution of matter.

Secondly. General formulas must be established for the motions of any kind of molecular system, which we conceive may exist *in rerum natura*.

Thirdly. We must determine as far as possible the kinds of molecular systems which are suited to the different primitive bodies; and be prepared to make other applications suitable for the explanation of phenomena.

Of these three things, the first, which is the very foundation of molecular mechanics, can, I think, be done at once. The second also, though it requires a larger treatment, will not present any great difficulty. The third, however, in this first attempt, can be but very imperfectly accomplished; for sciences also have their infancy, nor am I so bold as to expect to be able to do what requires the labour of many: I shall only say so much as may suffice to establish for this science a definite existence and a proper form.

In order to give an idea of my plan, I will now say a few words on each of these three points.

I. PRINCIPLES OF MOLECULAR MECHANICS.

First, then, (to say nothing of the name of “*molecular mechanics*,” which will be justified later,) in all bodies we find these three things, *extension*, *inertia*, and *active powers*, to one or other of which every property of bodies may be referred. In order therefore to arrive at a clear idea of the constitution of natural substances, these three must be diligently investigated.

Extension.—I have come to the following conclusions on this head, which,

I think, can be established by evident arguments drawn from various considerations.

1. All bodies consist of simple and unextended elements, the sum of which constitute the *absolute mass* of the given body. The extension itself, or *volume*, of the body is nothing but the extension of the space included within the bounding surfaces of the body; and the *extension of space* is nothing but its capability of being passed through (*percurribilitas*) in any direction by means of motion extending from any one point to any other.

2. There is no such thing possible as matter materially and mathematically continuous—that is to say, such that its parts touch each other with true and perfect contact. There must be admitted indeed a continuity of forces ready to act; but this continuity is only virtual, not actual nor formal.

3. Simple elements cannot be at once attractive at greater, and repulsive at less distances. To this extent at least Boscovich's theory must be corrected. If an element is attractive at any distance, it will be so at all distances; and if it be repulsive at any distance, it will be repulsive at all distances. This is proved from the very nature of matter, and perfectly corresponds with the action of molecules and with universal attraction.

4. Simple *elements* must not be confounded with the *atoms* of the chemist, nor with the molecules of which bodies are composed. Molecules are, according to their name, small extended masses, *i. e.* they imply volume; elements are indivisible points without extension. Again, molecules of whatever kind, even those of primitive bodies, are so many systems resulting from elements acting on each other; consequently elements differ from molecules as parts differ from the whole; so that much may be said about separate *elements*, which cannot be said of separate molecules or chemical atoms, and *vice versâ*. *Element, molecule, body* have the same relation to each other in the physical order, that *individual, family, state* bear to each other in the social order; for a body results from molecules, and molecules from elements holding together mechanically, in a similar way to that in which a state results from families, and families from individuals bound together by social ties.

So much regarding *extension*; for I do not now intend to proceed to the demonstration of these statements, but simply to put down what it is I am prepared to prove.

Inertia.—There would scarcely be any need of saying anything on this head, were there not some, even learned men, who entertain false ideas about it, and from not rightly understanding what is said of inertia by physical philosophers, throw out ill-founded doubts, which do more harm than good to science. I say, then,

1. Inertia implies two things: (*a*) that each element of matter is perfectly indifferent to receiving motion in any direction and of any intensity from some external agent; (*b*) that no element of matter can move itself by any action of its own.

2. It follows as a sort of corollary from this, that to be inert does not signify to be *without active power*; and that the very same element, which on account of its inertia cannot act upon itself, may, notwithstanding this inertia, have an active power, by which it may act upon any other element whatever.

3. Inertia is an essential property of matter, and is not greater in one element than in another, but is always the same in all elements, whether they are attractive or repulsive, whether their active power is great or small.

4. That which is called by natural philosophers the *vis inertia* is not a special mechanical force added on to the active forces of elements, but is the readiness of a body to react by means of its elementary forces, against any action tending to change the actual condition of that body.

These four propositions will remove many false notions, which give rise to confusion of ideas and impede the solution of many important questions.

Active power.—The questions relating to the active power of matter are of the greatest importance, since on them depends nearly the whole science of nature. On this point I am convinced, and think I can prove, that

1. No other forces exist in the elements of matter except *locomotive* or mechanical forces; for these alone are required, and these alone are sufficient, to account for all natural phenomena. So that we need have no anxiety about the *vires occultæ* of the ancients, nor need we make search after any other kind of *primitive* forces, besides such as are mechanical or locomotive. Hence chemical, electric, magnetic, calorific and other such actions will be all reduced to mechanical actions, complex indeed, but all following certain definite laws, and capable of being expressed by mathematical formulæ as in general mechanics. Hence in treating of *molecular mechanics* we do not make any gratuitous assumption or probable hypothesis, but are engaged on a branch of science founded on demonstrable truths, free from all hypothesis or arbitrary assumption.

2. There are not only attractive, but also repulsive elements; and this is the reason why molecules of bodies (as being made up of both sorts) may at certain distances attract, and at others repel each other.

3. Simple elements, in the whole sphere of their active power, and consequently *also at molecular distances*, act (whether by attracting or repelling) according to the inverse ratio of the squares of the distances. This proposition may seem to contradict certain known laws, as far as regards molecular distances; but the contradiction is only apparent, and this appearance will vanish when we consider that the action of *elements* (of which we are now speaking) is not the same as the action of *molecules*. From the fact that cohesion, *e. g.*, does not follow the inverse ratio of the square of the distance, it will certainly result that *molecules* do not act according to this law, and this is what physical science teaches: but it does not follow that *elements* do not act according to the law. This truth is, as all must see, of the utmost importance, since it is the foundation of molecular

mechanics, of which it would be impossible to treat at all, unless the law of elementary action at infinitesimal distances were known. This truth universalizes Newton's law of celestial attraction by extending it to all elementary action, whether attractive or repulsive, and makes it applicable not only to telescopic, but also to microscopic distances. It is clear therefore that I am bound to prove this law most irrefragably, lest I construct my molecular mechanics on an insecure foundation.

4. The sphere of the activity of matter is indefinite, in this sense, that no finite distance can be assigned at which the action of matter will be null. It by no means, however, follows from this that the force of matter has an infinite intensity.

5. The natural activity of each element of matter is exerted *immediately* on every other existing element at any distance, either by attracting or repelling, according to the agent's nature. Thus, *e. g.*, the action which the earth exerts on each falling drop of rain is exerted *immediately* by each element of the earth on each element of the water (notwithstanding the distance between them); it is not exerted through the material medium of the air, or of ether, or any other substance. The same must be said of the action of the sun on the planets. This proposition, however, it is evident, holds only for the simple action of the elements, *i. e.*, attractive or repulsive. For it is clear that complex actions causing vibratory motions, such as light or sound, are only transmitted through some vibrating medium. This conclusion is also of immense importance, because it solves a question much discussed by the ancients about the nature of action exerted on a distant body, and removes all scruples of philosophers on this head.

6. Bodies do not and cannot act by mathematical contact, however much our prejudices incline us to think the contrary; but *every* material action is *always* exerted on something at a distance from the agent.

7. There is another prejudice which I wish to remove, *i. e.* that one motion is the *efficient cause* of another motion. It is easily shown that this mode of speaking, though sometimes employed by scientific men, is incorrect, and ought to be abandoned, because it tends to the destruction of all natural science. Motion never causes motion, but is only a condition affecting the agent in its manner of acting. For all motion is caused by some agent giving velocity and direction; but the agent gives velocity and direction by means of its own active power, which it exerts differently according as it is found in different local conditions. Now these local conditions of the agent may be differently modified by the movement of the agent itself. The impact of bodies, the change of motion to heat, the communication of velocity from one body to another (always a difficult question), and other points of a like nature can only be satisfactorily explained by this principle.

These are the principal points that have to be discussed, defined, and demonstrated in order that molecular mechanics may be established on solid principles.

II. MATHEMATICAL EVOLUTION OF THESE PRINCIPLES.

After establishing principles, we must proceed to investigate the formulas of motion and of equilibrium, first between the elements themselves, then between the several systems of elements. The difficulties to be overcome in establishing the principles were chiefly philosophical: the difficulties which occur in the present part are mathematical, and can only be overcome by labour and patience.

As long as we confine ourselves to two elements, the mathematical formula expressing their motion is easily found. Thus, if there are two attractive elements of equal intensity, and if v be the action of one for a unit of distance in a unit of time, $2a$ the distance between them at the beginning of motion, x the space passed through by one in the time t , the equation of motion will be

$$t = \sqrt{\frac{2a}{v}} \left(\sqrt{x(a-x)} + a \cdot \text{arc tang} \sqrt{\frac{x}{a-x}} \right).$$

And since it is clear, from other considerations, that these two elements must vibrate together indefinitely in vibrations of equal times and constant extent, the time of one oscillation will easily be found from the above formula.

If the two attractive elements have unequal forces, or if one be attractive and the other repulsive, or both repulsive, the equation of motion may easily be obtained.

But when we have to do with a more complex system of elements, after obtaining the differential equations corresponding to the nature of the system, it is scarcely possible to obtain their integration, as will appear from the examples which I shall give below. Consequently, if we wish to deduce anything from such equations, we must proceed indirectly, and a long labour must be undertaken, sometimes with but slender results. This material difficulty will be diminished, or perhaps disappear, either by some new method of integration (which I can scarcely dare to hope for, though it is a great desideratum) or by certain tables exhibiting series of numerical values belonging to different systems.

But there occurs another difficulty in these systems. For since the agglomerations of simple elements can be arranged in an infinite variety, and it would be neither reasonable nor possible to treat of all such agglomerations, we must limit the number of them according to the scope we have in view, *i. e.* according to the use they may be of in explaining natural phenomena. Even this is a very difficult matter. How I have endeavoured to overcome this difficulty I will briefly explain.

First. I considered that the molecules of primitive bodies, such as oxygen, hydrogen, nitrogen, &c., cannot reasonably be supposed to be *irregular*—a conclusion which, though I cannot rigorously demonstrate, yet I can render probable by good reasons. Consequently, while treating of primitive systems I may confine myself to the examen of forms that are *regular*.

Secondly. I divided these regular systems into different classes according to their geometrical figure. Of these I have investigated the tetrahedric, octahedric, hexahedric, octohexahedric, pentagonal-dodecahedric, and icosahedric.

I then divided these classes into different species, viz. *pure centrata*, *centro-nucleata*, *centro-binucleata*, *centro-trinucleata*, &c., also into *acentrata* (without centre), *truncata*, &c. To enumerate the whole would take too long; indeed I only mention these to show how in such a multiplicity of systems I endeavoured to introduce the order necessary for me to be able to speak distinctly about them.

Lastly, besides classes and species, it was requisite also to consider certain distinct varieties under the same species. And in this way I seemed to myself to have embraced all the regular systems of elements possibly conceivable.

Thirdly. The several parts of which any system of elements can consist are reduced by me to a *centre*, *nuclei* to any number, and an external *envelope*. And thus I obtained not only a method of nomenclature for the different systems (a most important point), but also a method of exhibiting each system under brief and intelligible symbols. Thus, *e. g.*, the tetrahedric system *pure centratum* (*i. e.* without any nucleus), in which the centre is an attractive element, and the four elements of the envelope repulsive, will be represented thus,

$$m = A + 4R,$$

in which expression m signifies the absolute mass of the system (in this case $m=5$), A represents the attractive centre, and $4R$ the four repulsive elements of the envelope. The letters A and R are not *quantities*, but only *indices* denoting the nature of the action.

In a similar way, the following expression

$$m = R + 6A + 8R'$$

denotes a system whose centre R is repulsive, whose single nucleus $6A$ is octahedric and attractive, and whose envelope $8R'$ is hexahedric and repulsive: m , which, as before, indicates the absolute mass of the system, here = 15.

This will suffice to show how the different species and varieties of the afore-mentioned systems may be named and expressed.

Then I had to find mechanical formulas for the motion or equilibrium of the several systems; for it is only from such formulas that we can determine what systems are generally possible in the molecules of bodies. Speaking generally, no system *pure centratum*, of whatever figure it be, can be admitted in the molecules of natural bodies, whether gaseous, liquid, or solid.

Let v represent the action of the centre, and w that of one of the elements of the envelope for a unit of distance; and let r be the radius of the system, *i. e.* the distance of any one of the elements of the envelope from the centre;

the general formula of motion for any system *pure centratum* (expressed as above by $m=A+nR$) will be

$$\frac{d^2r}{dt^2} = -\frac{1}{r^2}(v-Mw),$$

where M signifies a constant, and the actions which tend to increase r are taken as positive.

If the system is tetrahedric,	$M=0.91856$
„ octahedric,	$M=1.66430$
„ hexahedric,	$M=2.46759$
„ octohexahedric,	$M=4.11170$
„ icosahedric,	$M=4.19000$
pentagonal dodecahedric,	$M=7.82419$.

Now none of these varieties satisfies the conditions either of solid, liquid, or gaseous bodies; because they either will not resist compression, or they form masses which are repulsive at all great distances; or if they could constitute gaseous bodies, they do not allow the law of compression to be verified, which we know to hold for all gases.

Passing on to the systems *centro-nucleata*, the formulas will differ according to the several figures of the nuclei and envelope. Taking, *e. g.*, the system

$$m=R+6A+8R',$$

which is hexahedric with an octahedric nucleus, and taking v, v', w to represent respectively the actions of the centre, one element of the nucleus, and one element of the envelope; taking also r and ρ for the radii of the nucleus and envelope, the equations of motion for such a system will be

$$\frac{d^2r}{dt^2} = \frac{v-M'v'}{r^2} + 4w \left(\frac{r+\rho\sqrt{\frac{1}{3}}}{\sqrt{\left(\rho^2+r^2+\frac{2\rho r}{\sqrt{3}}\right)^3}} + \frac{r-\rho\sqrt{\frac{1}{3}}}{\sqrt{\left(\rho^2+r^2-\frac{2\rho r}{\sqrt{3}}\right)^3}} \right),$$

$$\frac{d^2\rho}{dt^2} = \frac{v+Mw}{\rho^2} - 3v' \left(\frac{\rho+r\sqrt{\frac{1}{3}}}{\sqrt{\left(\rho^2+r^2+\frac{2\rho r}{\sqrt{3}}\right)^3}} + \frac{\rho-r\sqrt{\frac{1}{3}}}{\sqrt{\left(\rho^2+r^2-\frac{2\rho r}{\sqrt{3}}\right)^3}} \right),$$

where $M=2.46759$, and $M'=1.66430$. The conditions of equilibrium will be obtained by making the two first members equal to zero.

What systems of this class (*centro-nucleata*) can satisfy the conditions of solid, liquid, or gaseous bodies, is exceedingly difficult to determine, for reasons which I have above touched on, viz. that the formulæ of these systems are not integrable, and we have consequently to proceed indirectly with great expenditure of time and trouble. It seems to me, however, as far as I can judge, that some of these systems may be found in *rerum natura*.

Passing to another class of systems (*centro-binucleata*), we shall have three equations to express its laws of motion. Taking, *e. g.*, the system

$$m=A+4R+4A'+4R',$$

which is tetrahedric with two tetrahedric nuclei; taking v, v', v'', w for the respective actions of the elements acting from the centre, first and second nuclei, and envelope; taking r, r'', ρ for the radii of the two nuclei and the envelope, the equations of motion will be as follows:

$$\begin{aligned} \frac{d^2 r'}{dt^2} &= \frac{Mv' - v}{r'^2} - v'' \left(\frac{1}{(r' + r'')^2} + \frac{3r' - r''}{\sqrt{\left(r'^2 + r''^2 - \frac{2r'r''}{3}\right)^3}} \right) \\ &\quad - w \left(\frac{1}{(\rho - r')^2} - \frac{3r' + \rho}{\sqrt{\left(\rho^2 + r'^2 + \frac{2\rho r'}{3}\right)^3}} \right); \\ \frac{d^2 r''}{dt^2} &= \frac{Mv'' - v}{r''^2} + v' \left(\frac{1}{(r' + r'')^2} + \frac{3r'' - r'}{\sqrt{\left(r'^2 + r''^2 - \frac{2r'r''}{3}\right)^3}} \right) \\ &\quad + w \left(\frac{1}{(\rho + r'')^2} + \frac{3r'' - \rho}{\sqrt{\left(\rho^2 + r''^2 - \frac{2\rho r''}{3}\right)^3}} \right); \\ \frac{d^2 \rho}{dt^2} &= \frac{Mw - v}{\rho^2} + v' \left(\frac{1}{(\rho - r')^2} + \frac{3\rho + r'}{\sqrt{\left(\rho^2 + r'^2 + \frac{2\rho r'}{3}\right)^3}} \right) \\ &\quad - v'' \left(\frac{1}{(\rho + r'')^2} + \frac{3\rho - r''}{\sqrt{\left(\rho^2 + r''^2 - \frac{2\rho r''}{3}\right)^3}} \right); \end{aligned}$$

in which equations $M=0.91856$.

The discussion of these equations and similar ones will afford a useful occupation to mathematicians and natural philosophers. Whatever conclusions may be drawn from them cannot fail to throw great light on the question of the nature of bodies.

It is evident that we might go further and pass on to *trinucleate*, *quadrinucleate*, &c. systems; but the number of equations will increase in proportion, together with the difficulty of dealing with them.

It is not enough to consider the laws of motion and equilibrium in each system separately, but it is also necessary to know what action one system exercises on another, whether like or unlike, placed at a given distance. For since many of the properties of bodies depend on the relation which the different molecules bear to one another, *e. g.*, liquidity, elasticity, hardness, &c., it is not enough to know what is the state of a system of elements (*i. e.* a molecule) in itself, but we must investigate also how several such systems (or molecules) affect each other. Now in this ulterior investigation it is clear that the difficulty increases exceedingly, since the equations become exceedingly complex. Here also then may natural philosophers

find matter for industry and patience. I have done a little in this subject, but not enough to deserve any special mention. In order, however, to diminish the difficulties, the investigation may be provisionally restricted to the mutual actions of the *envelopes*, neglecting for the time that of the *nuclei*, which may be considered as a disturbing cause, for which some correction may afterwards have to be made.

So much then for the mathematical and theoretic development of molecular mechanics. There remains the third part, which, though the most laborious of all, will yet give the greatest pleasure to scientific men; since it is less dry, and opens a way for attaining the end aimed at in the natural sciences. Of this third part I will add a few words.

III. APPLICATION OF THE PRINCIPLES OF MOLECULAR MECHANICS.

[Under this head the author points out the various properties of bodies which would have to be explained, and of which he conceives an explanation might be afforded could the mathematical calculations be effected which are required for the elaboration of his theory, and enunciates the following conclusions as deduced from his explanation of the impact of bodies.]

1. If a body does not contain any repulsive elements, it cannot cause any retardation in the movement of any impinging body.

2. Again, if the medium through which a body moves contain no repulsive elements, no retardation of its motion can take place.

3. If a medium does contain repulsive elements, retardation must necessarily take place.

4. Consequently, as the planets in their movements through the æther do not suffer any loss of velocity, it must be concluded that the æther does not contain any repulsive elements at all, and that its elasticity must be explained without any recourse to repulsive forces.

This last inference is somewhat wonderful, and decidedly curious: but after much consideration it appeared to me so natural, and so well harmonizing with other truths and scientific theories, that I ceased to hesitate about its adoption and gave it a most decided assent; whether wisely or not, I leave others to judge.

III. "On some further Evidence bearing on the Excavation of the Valley of the Somme by River-action, as exhibited in a Section at Druat near Abbeville." By JOSEPH PRESTWICH, F.R.S.
Received January 29, 1864.

On the occasion of a late visit to Abbeville, I noticed a fact which appears of sufficient interest, as bearing upon and confirming one of the points treated of in my last paper, to induce me to submit a short notice of it to the Royal Society. It occurs in a tributary valley to that of the Somme, but necessarily forms part of the general phenomena affecting the whole basin.